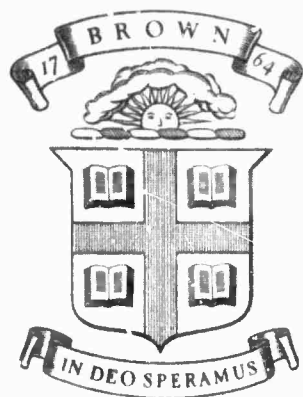


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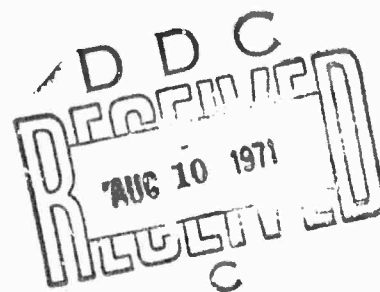


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UNDRAINED PLANE-STRAIN EXPANSION  
OF A CYLINDRICAL CAVITY IN CLAY:  
A SIMPLE INTERPRETATION OF THE  
PRESSUREMETER TEST

ANDREW C. PALMER

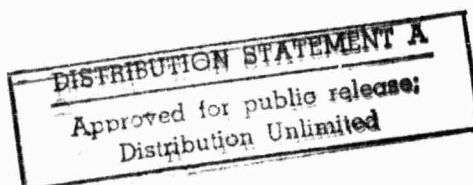
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UNDRAINED PLANE-STRAIN EXPANSION OF A CYLINDRICAL CAVITY IN CLAY:  
A SIMPLE INTERPRETATION OF THE PRESSUREMETER TEST

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July 1971

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## ABSTRACT

A study is made of the problem of interpreting the results of Menard pressuremeter tests on soil in situ. In the test a cylindrical cavity is expanded by internal pressure, and the relation between applied pressure and cavity volume change is measured. It is shown that the results of a single test are enough to determine a complete stress-strain relation in plane strain, and a simple graphical procedure for doing this is derived. The only restrictive assumption necessary is that the deformation is under undrained conditions; there is no restriction to infinitesimal strain or to elastic perfectly-plastic soils.

## INTRODUCTION

The Ménard pressuremeter is an instrument designed to measure the shear strength of soil in situ, without bringing a sample to the surface. The difficulties of obtaining undisturbed samples are then avoided. The instrument and its use have been described by Ménard (1957,1961) and Gibson and Anderson (1963). It is illustrated schematically in Figure 1. It consists of three cylindrical rubber cells, coaxial and equal in diameter, which are lowered into a previously-drilled borehole of relatively small diameter. Once in position, the central measuring cell is loaded by a steadily increasing internal pressure, which pushes the cylindrical surface of the cell outwards against the wall of the borehole, and what is measured is the relation between the applied pressure and the increase in cell volume. The two outer 'guard cells' are subjected to the same internal pressure as the measuring cell, but their volume changes are not measured: their function is to eliminate end effects and ensure a state of plane deformation in the region surrounding the measuring cell. Corrections are made for the pressure difference required to inflate the rubber cell itself and for the volume increase of the connecting pressure leads.

If the measurements made in the test are to be useful, a method has to be found for deriving from the measured pressure-volume relation some numerical parameters describing the stress-strain relation of the soil, such, for instance, as the shear strength and an elastic stiffness modulus. The existing work on this question relies on idealized models of soil properties. Ménard (1957) supposed the soil to be elastic perfectly-plastic, and examined both the case in which deformations are small enough for geometry change effects to be neglected and the case when deformations are very large, but not the intermediate case. He rederived an expression previously obtained by Bishop, Hill and Mott (1945) for the limit pressure at which a cylindrical cavity in an elastic perfectly-plastic medium

can continue to expand indefinitely. He also gave some attention to the case in which the soil is not perfectly-plastic, but did not carry this analysis through. Gibson and Anderson (1963) included geometry change effects in their analysis, but again supposed the soil to be elastic perfectly-plastic. They derived a complete relation between imposed pressure and cavity volume change under these conditions, and suggested a way of calculating the yield stress.

It will be shown in this paper that more can be learned from the test if fewer assumptions are made. The most important change is the dropping of the assumption that the soil is perfectly plastic, an assumption which is in any case inappropriate for many soils. Contrary to what one might expect, the analysis is simpler without this assumption, and it is then possible to derive a complete stress-strain relation for plane strain deformation of the soil from the results of a single pressuremeter test. The paper begins with a theoretical analysis, and continues to a reinterpretation of the results reported by Gibson and Anderson.

## ANALYSIS

The kinematics of deformation are considered first. The analysis is restricted to undrained deformation of a saturated soil, and therefore corresponds to a pressuremeter test carried out relatively quickly, in a time short compared to drainage times for distances of the order of the borehole radius. This is almost invariably the case when the pressuremeter is used in clay, but would not be appropriate for a highly permeable sandy soil, where internal drainage during a test will modify the undrained condition. The corresponding drained problem is more difficult, and requires stronger assumptions about the stress-strain relation of the soil. A solution for the special case of normally-consolidated clay has

been given by Palmer and Mitchell (1971).

When the borehole is made, the reduction to zero in the radial stress at the borehole wall induces a change in stress in the soil surrounding the hole. If this change is not too large, the response of the soil around the hole will be elastic and reversible. When the pressuremeter is placed in the hole and the measuring cell pressure is increased, the soil around the hole will be reloaded. When the pressure applied reaches the horizontal stress  $\sigma_h$  originally present in the soil, all the soil surrounding the hole will have returned to its initial condition, since the unloading was reversible. It is natural to take this as the reference state, and to refer all displacements and strains to this state. All lengths and displacements will be non-dimensionalized with respect to the radius of the borehole in the reference state.

By the conditions of the test, the deformation is in plane strain, undrained, and axially symmetric about the pressuremeter axis. At each point in the soil, the principal directions are the local radial, axial and circumferential directions. All displacements are radial. In the reference state, the borehole radius is 1 and a typical material point in the soil is at a radius  $r-y$ . At some later time in the test the borehole radius has increased from 1 to  $1 + y_1$ , and the material point initially at radius  $r - y$  has moved outward to radius  $r$ , so that  $y$  is the displacement of the point which has reached  $r$  in the deformed state. This is illustrated schematically in Figure 2. Since the deformation is undrained

$$r^2 - (1 + y_1)^2 = (r - y)^2 - 1 \quad (1)$$

whence

$$y = r - \left[ r^2 - y_1(2 + y_1) \right]^{1/2} \quad (2)$$

Define the circumferential principal extension  $e_\theta$  as the increase in length of a material line element initially in the circumferential direction divided by its length in the reference state. Since principal axes do not rotate, no more sophisticated definition of extension is necessary. Then

$$e_\theta = \frac{y}{r-y} = -1 + \left[ 1 - \frac{y_1(2+y_1)}{r^2} \right]^{-1/2} \quad (3)$$

and is positive. Since the axial extension is zero, and there is no volume change, the radial principal extension  $e_r$  is related to  $e_\theta$  by

$$(1 + e_\theta)(1 + e_r) = 1 \quad (4)$$

It is convenient to introduce here the volume increase ratio  $\Delta V/V$  defined by Gibson and Anderson, the increase in volume of the measuring cell divided by the volume in the deformed state (not by the volume in the reference state). By this definition

$$\Delta V/V = \frac{(1 + y_1)^2 - 1}{(1 + y_1)^2} \quad (5)$$

$$y_1 = (1 - \Delta V/V)^{-1/2} - 1 \quad (6)$$

The conditions imposed by equilibrium are considered next. In the reference state the radial and circumferential stresses are equal and uniform, and equal to the horizontal stress  $\sigma_h$  at a large distance from the borehole. As the radius of the hole is increased, the circumferential extension  $e_\theta$  is positive and the radial extension  $e_r$  is negative. The corresponding shear strain induces a difference between the circumferential and radial effective stresses  $\sigma'_\theta$  and  $\sigma'_r$ , a difference which is a function of the difference in principal strains and there-

fore a function of  $e_\theta$ . Call this latter function  $\phi(e_\theta)$ . In studies of finite deformation more than one distinct definition of stress is possible (see, for example, Truesdell (1965)), but the stress referred to here is the Cauchy stress; sometimes called the true stress. Accordingly

$$\sigma_r' - \sigma_\theta' = \phi(e_\theta) \quad (7)$$

The radial equilibrium equation is

$$r \frac{d\sigma_r}{dr} = \sigma_\theta - \sigma_r = \sigma_\theta' - \sigma_r' \quad (8)$$

where  $\sigma_\theta$  and  $\sigma_r$  are total stresses. Substituting from equation (3) into (7) and then into (8)

$$\frac{d\sigma_r}{dr} = \frac{1}{r} \phi \left( -1 + \left[ 1 - \frac{y_1(2 + y_1)}{r^2} \right]^{-1/2} \right) \quad (9)$$

At the inside boundary  $\sigma_r$  is equal to the applied pressure, which is a measured function  $\psi$  of the radial displacement  $y_1$  calculated from  $\Delta V$  and equation (6). As  $r \rightarrow \infty$ ,  $\sigma_r$  tends to  $\sigma_h$ , the total stress far away from the borehole. Therefore, integrating (9) from the hole boundary at  $r = 1 + y_1$  to infinity,

$$\psi(y_1) - \sigma_h = \int_{1+y_1}^{\infty} \frac{1}{r} \phi \left( -1 + \left[ 1 - \frac{y_1(2 + y_1)}{r^2} \right]^{-1/2} \right) dr \quad (10)$$

In the test  $\psi$  is measured. To determine a stress-strain relation, equation (10) must be solved for the unknown function  $\phi$ . Transform the integration variable  $r$  to  $x$ , defined by

$$x = -1 + \left[ 1 - \frac{y_1(2 + y_1)}{r^2} \right]^{-1/2} \quad (11)$$

Equation (10) then becomes

$$\psi(y_1) - \sigma_h = \int_0^{y_1} \frac{1}{(1+x)^3} \left(1 - \frac{1}{(1+x)^2}\right)^{-1} \phi(x) dx \quad (12)$$

Differentiate with respect to  $y_1$

$$\psi'(y_1) = \frac{1}{(1+y_1)^3} \left(1 - \frac{1}{(1+y_1)^2}\right)^{-1} \phi(y_1) \quad (13)$$

where  $\psi'(y_1)$  is the derivative of  $\psi$  with respect to its argument. Rearranging

$$\phi(y_1) = y_1(1+y_1)(2+y_1)\psi'(y_1) \quad (14)$$

Since  $\psi$  is a measured function of  $y_1$ , the unknown function  $\phi$  can easily be determined from equation (14). The right-hand side of this equation can be rewritten in a particularly convenient form in terms of the volume increase ratio  $\Delta V/V$ , suggested by Gibson and Anderson and defined by equation (5). After some manipulation (14) becomes

$$\phi(y_1) = 2 \frac{d\psi}{d(\log(\Delta V/V))} \quad (15)$$

The stress difference  $\phi$  required to produce a principal extension  $e$  is therefore twice the gradient of a graph of  $\psi$ , the pressure applied by the measuring cell, against  $\log(\Delta V/V)$ . An inflection in this graph corresponds to a peak in the stress-strain curve. The stress difference at large deformations, a residual strength, corresponds to twice the slope of the graph as  $\Delta V/V$  becomes large.

The test can also be used to define a shear modulus  $G$  for small strains from the reference state; this can be interpreted as an elastic modulus. Such a modulus is conventionally defined as the derivative of shear stress with respect to engineering shear strain, evaluated at zero strain, and is

$$G = \frac{1}{2} \left( \frac{d\phi}{dy_1} \right)_{y_1=0} = \left[ \psi'(y_1) \right]_{y_1=0} \quad (16)$$

from equation (14). It may sometimes be convenient to define a "Young's modulus"  $E$ , using the well-known relation

$$E = 2(1 + \nu)G \quad (17)$$

and estimating a Poisson's ratio  $\nu$ , but if this is done it must be noted that equation (17) is only applicable to an elastically isotropic material.

#### REINTERPRETATION OF TESTS REPORTED BY GIBSON AND ANDERSON

Gibson and Anderson (1963) report tests carried out in overconsolidated London clay at Bradwell, Essex, England, at an elevation of - 13 m (-43 ft) O.D. The soil conditions at the site have been carefully analyzed by other workers (Skempton, 1961; Skempton and LaRochelle, 1965) using independent methods, and in particular Skempton (1961) has estimated the horizontal stress  $\sigma_h$  initially present in the soil. This is especially helpful here, since in the analysis above changes are measured from a reference state in which the stress around the borehole is exactly as it was before the hole was bored.

Skempton estimates the horizontal effective stress  $\sigma_h'$  at vertical intervals of 3.05 m (10 ft). Interpolation for - 13 m O.D. gives  $\sigma_h' = 330 \text{ kN/m}^2$  (47.6 lb/in<sup>2</sup>).

The ground surface at the site is at + 1.8 m O.D. (+ 6 ft); the ground water level was at + 0.6 m (+ 2 ft) O.D., and a piezometer at 5.8 m (19 ft) below the surface indicated a pore pressure exactly corresponding to ground water level. The gauge measuring the pressure in the water-filled measuring cell system is at ground level. The total stress applied by the measuring cell to the ground is therefore the measured pressure plus the hydrostatic head due to the height difference between the pressure gauge and the cell in the ground. The initial total stress in the ground is  $\sigma_h'$  (evaluated above) plus the pore water pressure, which is here the difference between the ground water level and the measuring cell level. It follows that the total stress around the borehole will return to its initial value when the measured pressure equals  $\sigma_h$  minus the hydrostatic head corresponding to the height difference between the pressure gauge and the ground water level. If the pressure gauge is 0.6 m (2 ft) above the ground, this height difference is 1.8 m (6 ft), corresponding to  $18 \text{ kN/m}^2$  ( $2.6 \text{ lb/in}^2$ ), and the measured cell pressure at which the total stress in the ground is  $\sigma_h$  is  $330 - 18 = 312 \text{ kN/m}^2$  ( $47.6 - 2.6 = 45.0 \text{ lb/in}^2$ ). Volume changes  $\Delta V$  should therefore be measured from this datum. Gibson and Anderson used a different datum, and refer  $\Delta V$  to an initial state with zero cell pressure, though in their subsequent calculations they make an elastic correction for the pressure to restore the radial stress to  $\sigma_h$ . In Figure 3 their data is replotted, using a new datum at which the cell pressure is  $312 \text{ kN/m}^2$  ( $45 \text{ lb/in}^2$ ) and plotting the cell pressure  $\psi$  against  $\Delta V/V$  on a logarithmic scale. Since the curvature of the graph is not large, its slope  $d\psi/d(\log(\Delta V/V))$  can easily be found graphically. For any value of  $\Delta V/V$  equation (15) gives the stress difference  $\phi$  and equation (6) the corresponding principal extension. The stress difference is plotted in Figure 4 as a function of the principal extension. There is an inflection in the curve of Figure 3, and this corresponds to a peak in

the stress-strain curve at a stress difference of  $500 \text{ kN/m}^2$  ( $72 \text{ lb/in}^2$ ), and therefore a peak shear strength of  $250 \text{ kN/m}^2$  ( $36 \text{ lb/in}^2$ ). Working from the same data, but assuming perfect plasticity, Gibson and Anderson estimated the yield shear stress as  $210 \text{ kN/m}^2$  ( $30.7 \text{ lb/in}^2$ ), which corresponds to a principal stress difference of  $420 \text{ kN/m}^2$  ( $61.4 \text{ lb/in}^2$ ).

It is not possible to make a direct comparison with published independent observations of stress-strain relations in plane strain in clay from the same site at the same depth. Skempton (1961) quotes the results of triaxial tests on the same soil at the same depth, and they indicate shear strengths of the order of  $140 \text{ kN/m}^2$  ( $20 \text{ lb/in}^2$ ), smaller by a factor of 1.8. Gibson and Anderson suggested that this could be ascribed to sample disturbance. Skempton and LaRoche (1965, Figure 10) do however quote several graphs of deviatoric stress against axial strain in three tests on weathered London clay from the same site but from a smaller depth, 7.3 m (24 ft) below ground level. All three show similar results. As the axial strain increased, the deviatoric stress increased rapidly to a peak, and then fell to some 80 per cent of its peak value as the strain increased to 0.1. Peak shear stresses in the three tests occurred at axial compressive strains of approximately 0.018, 0.020 and 0.030, which are of the same order of magnitude as the strains to peak stress observed in the pressuremeter. Since the clay is weathered in one case and unweathered in the other, this agreement must be interpreted with caution, but is perhaps significant.

In Figure 5  $\psi$  is plotted against  $y_1$ . The graph is straight to  $\psi=0$ , when no pressure is being applied to the borehole, and this is consistent with the assumption that the unloading when the hole is made is elastic and reversible. Using equation (16) and the measured slope at  $y_1=0$ ,  $G=1.0 \times 10^4 \text{ kN/m}^2$  ( $1450 \text{ lb/in}^2$ ).

## THE EFFECT OF THE INITIAL STATE OF STRESS IN THE GROUND

In the analysis described earlier, volume changes are measured from a reference state in which the pressure applied by the measuring cell to the ground is equal to the initial horizontal stress before the borehole was made. The re-evaluation of Gibson and Anderson's measurements at the Bradwell site was able to draw on Skempton's careful analysis of the horizontal stress in the ground. Such an analysis will not usually be available, and it is natural to ask whether a mistaken or inaccurate estimate of the horizontal stress would seriously affect the results. The estimated value of  $\sigma_h$  corresponded to  $\psi=312 \text{ kN/m}^2$  ( $45 \text{ lb/in}^2$ ). The analysis has been repeated for a reference state at  $\psi=200 \text{ kN/m}^2$  ( $29 \text{ lb/in}^2$ ), which is close to the overburden pressure. The resulting stress-extension relation is plotted in Figure 6. Comparison with Figure 4 indicates that the calculated stress-extension relation is not, in this instance at least, strongly sensitive to errors in estimates of  $\sigma_h$ .

## DISCUSSION

A suggested procedure for the interpretation of results of pressuremeter tests in saturated clay, under undrained conditions, is described below.

1. Estimate the original value  $\sigma_h$  of the horizontal total stress in the ground at the depth of the instrument.
2. At each stage of the test, determine the change in volume  $\Delta V$  of the measuring cell, measured from a reference state in which the cell pressure is  $\sigma_h$ , and the current volume  $V$  of the cell.
3. Plot the cell pressure  $\psi$  against  $\log(\Delta V/V)$ . At each value of  $\Delta V/V$ , the slope of this graph is the principal stress difference corresponding to the <sup>twice</sup>

principal extension  $(1 - \Delta V/V)^{-1/2} - 1$  . The shear modulus for small strains is found by plotting  $\psi$  against the principal extension and evaluating the slope of this latter plot at zero principal extension.

The resulting stress-extension relation is appropriate to plane strain deformation in which the principal stress is vertical. Further interpretation may be necessary if it is to be used in circumstances in which the intermediate principal stress is horizontal.

#### NOTATION

$e_\theta$	circumferential extension
$e_r$	radial extension
$E$	Young's modulus
$G$	shear modulus
$r$	radius
$x$	integration variable, defined by equation (11)
$y$	radial displacement
$y_1$	radial displacement at borehole wall
$\nu$	Poisson's ratio
$\sigma_h$	original horizontal stress in the ground
$\sigma_r$	radial stress
$\sigma_\theta$	circumferential stress
$\phi$	difference between extreme principal stresses
$\psi$	pressure applied to borehole wall

Effective stresses are indicated by superscript primes. Following the usual convention of soil mechanics, compressive stresses are taken as positive.

## ACKNOWLEDGEMENT

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## REFERENCES

- BISHOP, R. F., HILL, R. and MOTT, N. F., 1945. 'The theory of indentation and hardness tests'. Proceedings of the Physical Society, 57 : 321 : 147-159.
- GIBSON, R. E. and ANDERSON, W. F., 1963. 'In-situ measurements of soil properties with the pressuremeter'. Civil Engineering and Public Works Review, 56 : 656 : 615-618.
- MENARD, L., 1957. 'Mesures in-situ des propriétés physiques des sols'. Annales des Ponts et Chaussées, 127 : 14 : 357-377.
- PALMER, A. C. and MITCHELL, R. J., 1971. 'Plane-strain expansion of a cylindrical cavity in clay'. University of Cambridge, Department of Engineering, Technical Report CUED/C-SOILS/TR3 ; to appear in the Proceedings of the Roscoe Memorial Symposium, Cambridge, 1971.
- SKEMPTON, A. W., 'Horizontal stresses in an over-consolidated Eocene clay'. Proceedings, Fifth International Conference on Soil Mechanics, 1 : 351-357.
- SKEMPTON, A. W. and LAROCHELLE, P., 1965. 'The Bradwell slip : a short-term failure in London clay'. Geotechnique, 15 : 3 : 221-242.
- TRUESDELL, C., 1966. 'The elements of continuum mechanics'. Springer-Verlag, New York.

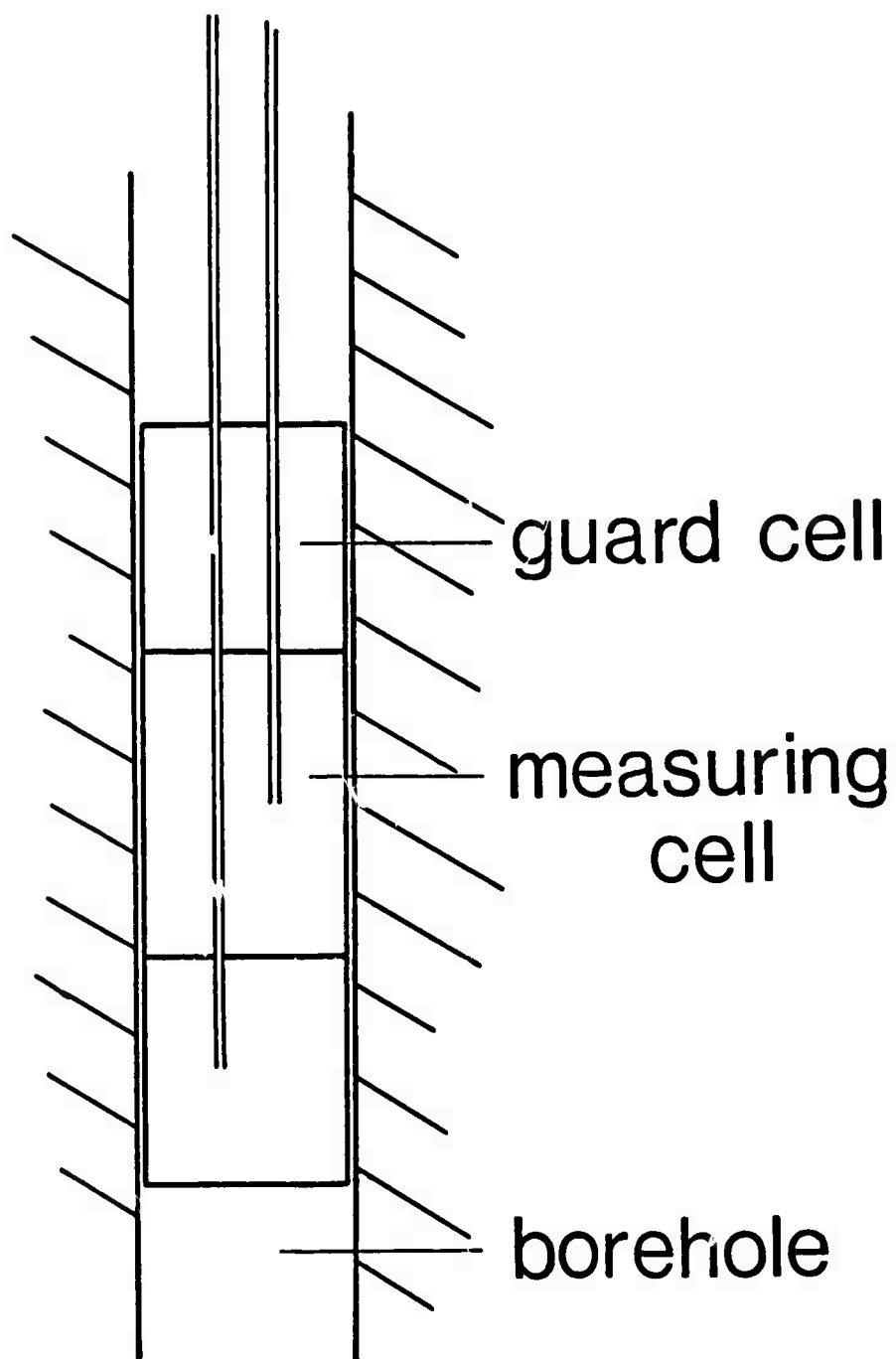
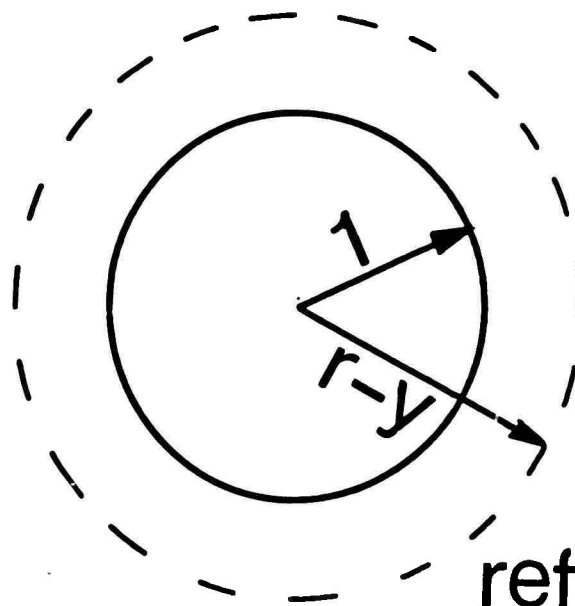
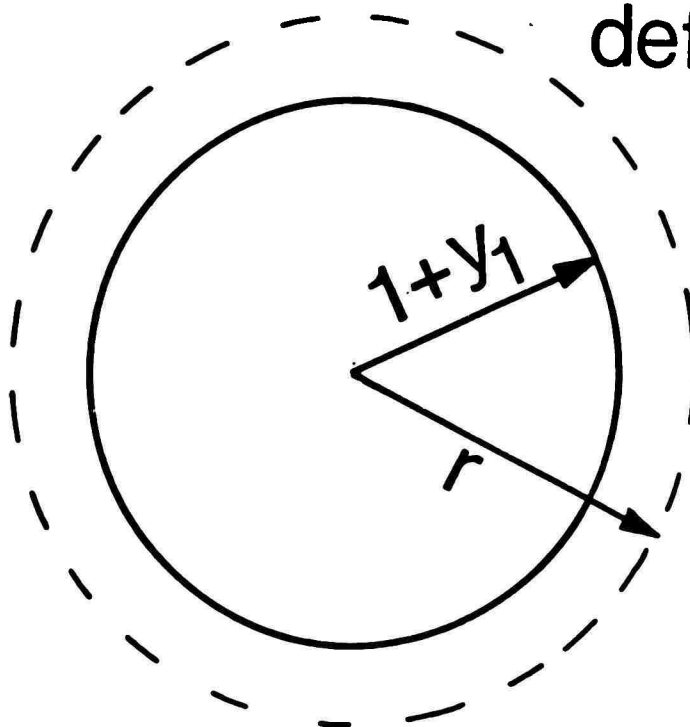


FIGURE 1



reference  
state



deformed  
state

FIGURE 2

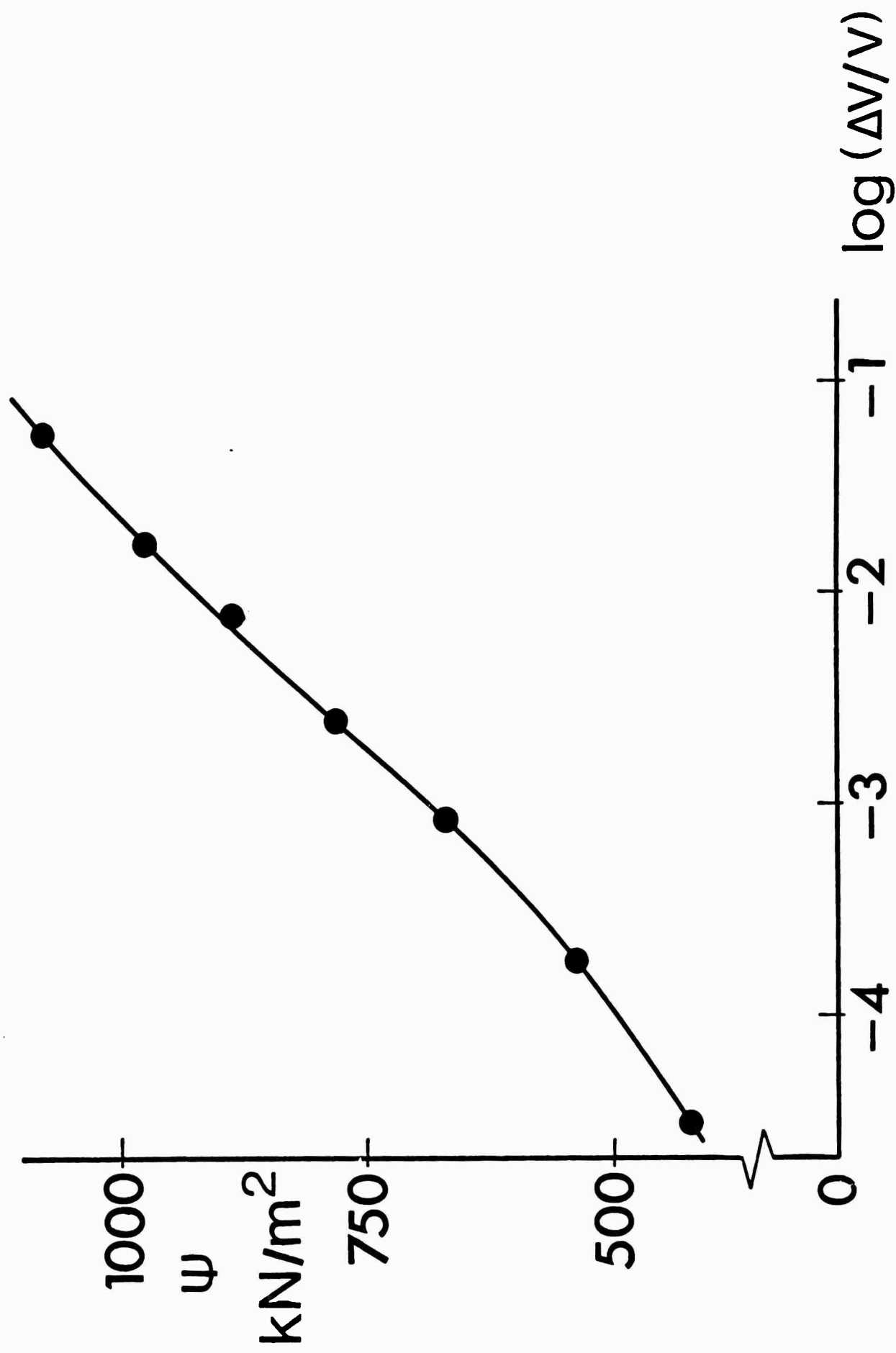


FIGURE 3

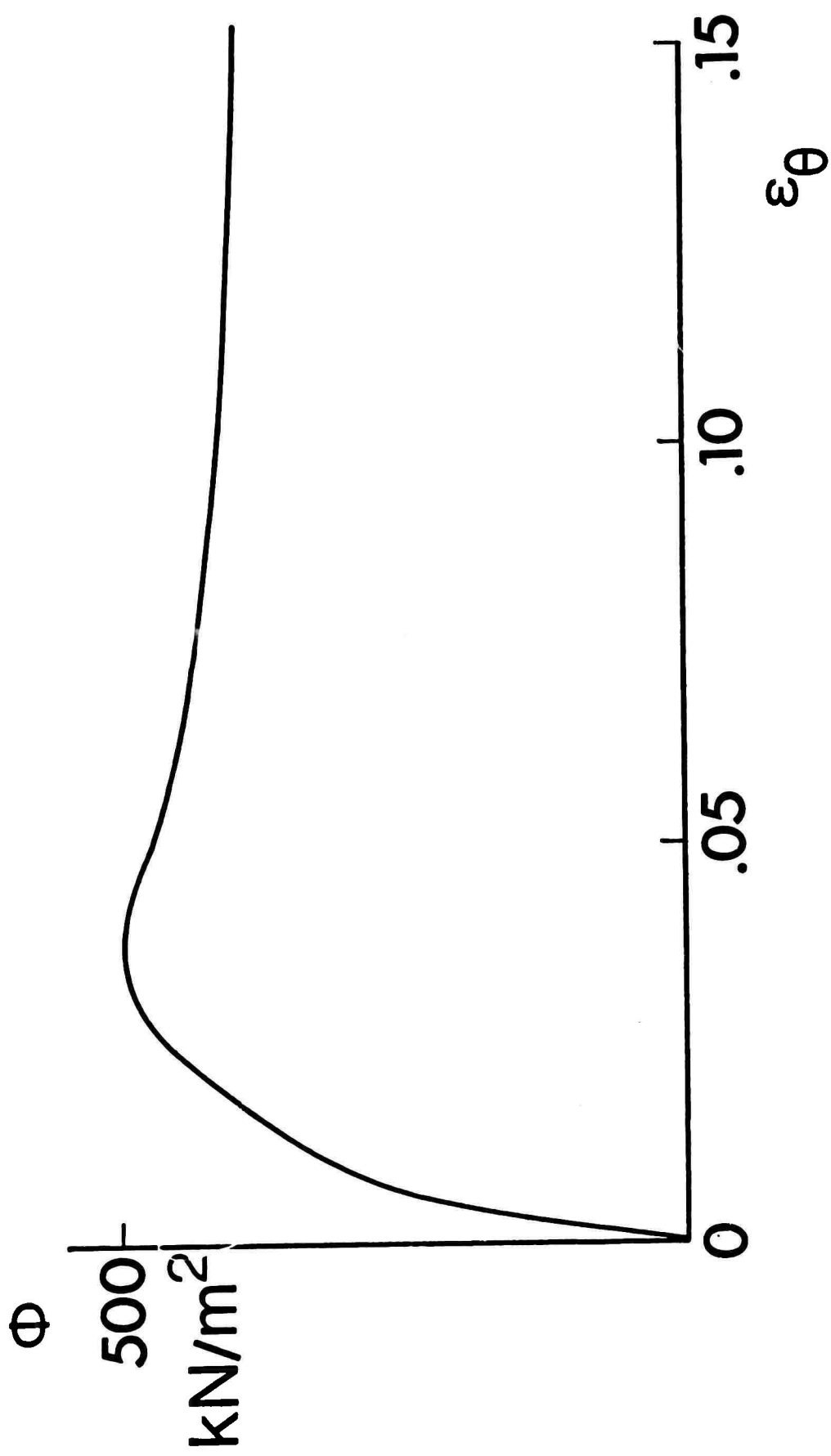


FIGURE 4

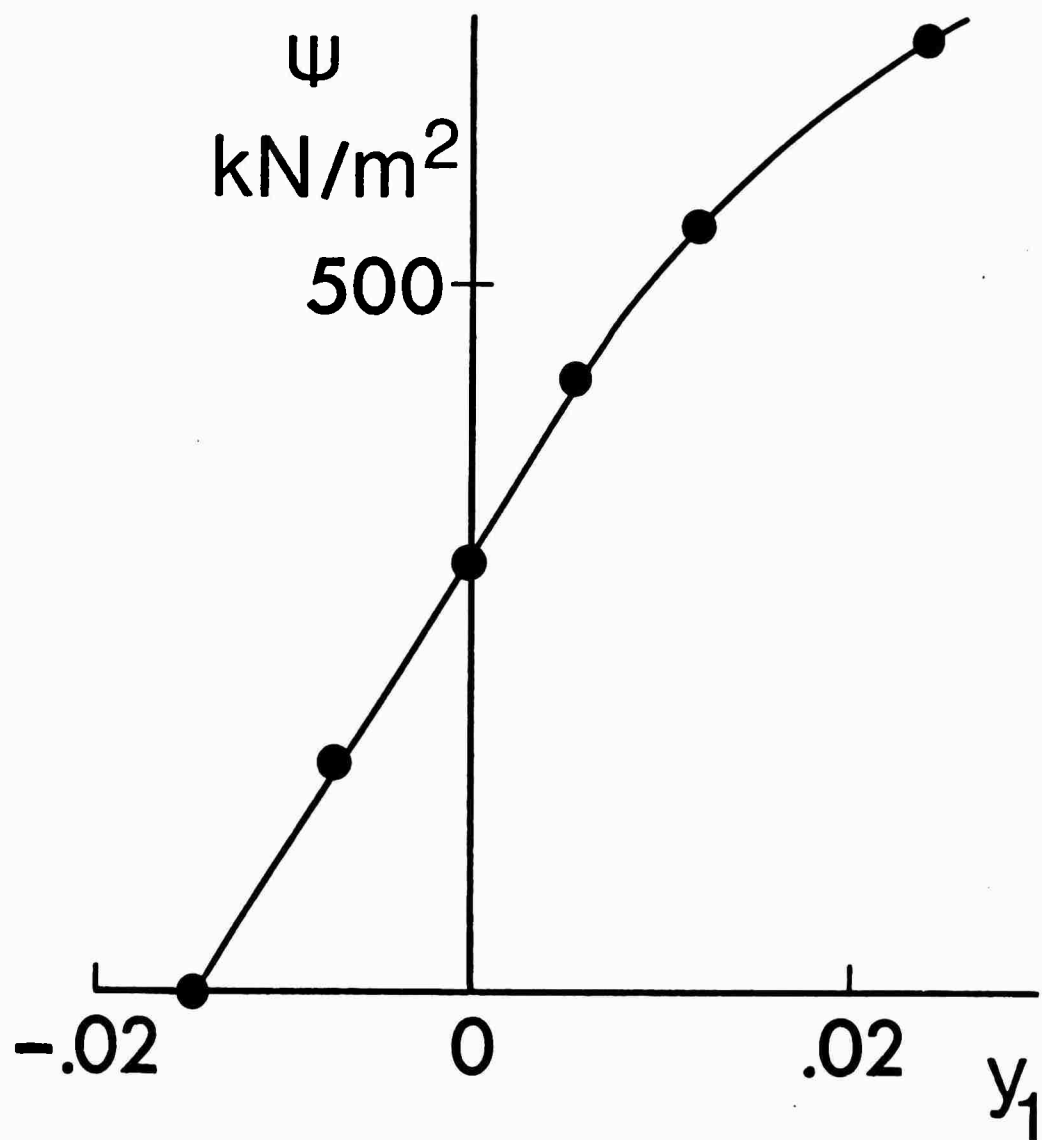


FIGURE 5

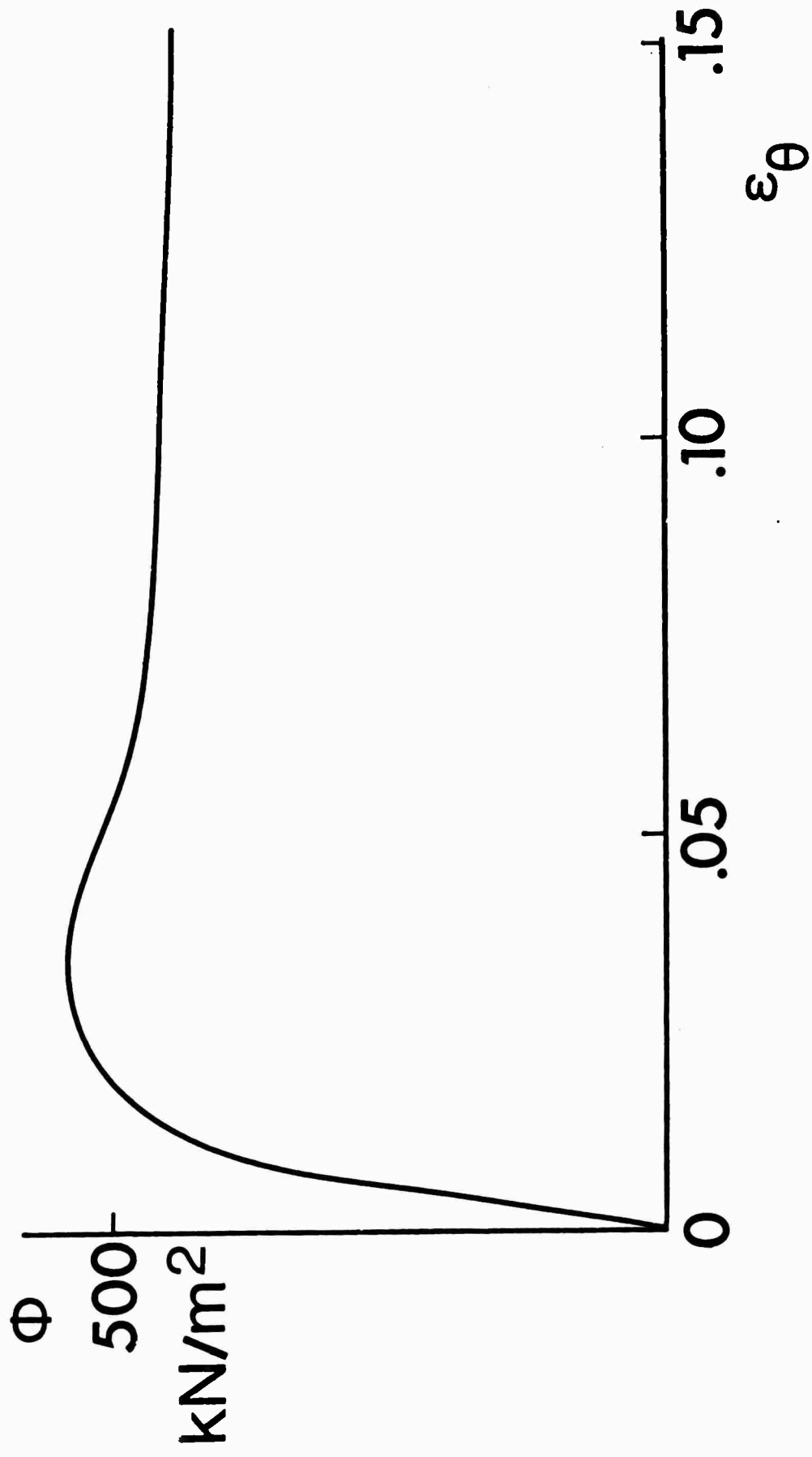


FIGURE 6